Progress in Nonlinear Differential Equations
and Their Applications

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Ginzburg-Landau Vortices

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This book is concerned with the study in two dimensions of stationary solutions u_ε of a complex valued Ginzburg-Landau equation involving a small parameter ε. Such problems are related to questions occurring in physics: e.g., phase transition phenomena in superconductors and superfluids. The parameter ε has a dimension of a length which is usually small. Thus, it is of great interest to study the asymptotics as ε tends to zero.

One of the main results asserts that the limit $u_\star$ of minimizers u_ε exists. Moreover, $u_\star$ is smooth except at a finite number of points called defects or vortices in physics. The number of these defects is exactly the Brouwer degree — or winding number — of the boundary condition. Each singularity has degree one — or as physicists would say, vortices are quantized.

The singularities have infinite energy, but after removing the core energy we are led to a concept of finite renormalized energy. The location of the singularities is completely determined by minimizing the renormalized energy among all possible configurations of defects.

The limit $u_\star$ can also be viewed as a geometrical object. It is a minimizing harmonic map into S^1 with prescribed boundary condition g. Topological obstructions imply that every map u into S^1 with $u=g$ on the boundary must have infinite energy. Even though $u_\star$ has infinite energy, one can think of $u_\star$ as having "less" infinite energy than any other map u with $u=g$ on the boundary.

The material presented in this book covers mostly recent and original results by the authors. It assumes a moderate knowledge of nonlinear functional analysis, partial differential equations, and complex functions. This book is designed for researchers and graduate students alike, and can be used as a one-semester text.

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